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CALCULATION AND DESIGN OF RADIATORS FOR THE AIR COOLING SYSTEM

OF A GROUP OF INSTRUMENTS

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The article suggests a method of selecting the cooling regime and of planning the design of a radiator for a group of electronic instruments.

1. Thermal Model and Analysis of the Efficiency of the Radiators. Various radiators have found widespread application in electronics, radio engineering, electrical engineering, and other branches of instrument making for the purpose of cooling thermally loaded elements.

In design these devices differ according to the kind of developed surface [1]: lamellar, finned, pin type, "crab" type, louver type, and wire loop. The magnitude of the scattered power is substantially affected by the following geometric parameters: dimensions of the base ( $L_x$ ,  $L_y$  for rectangular radiators; D - the diameter - for circular radiators), height H and thickness d of the fin or pin, and pitch S between them. For wire-loop radiators we have to take into account the height of a turn H, the wire diameter d, the pitch of cooling S, the spacing of the coils S', and the space factor of the channel  $\varphi$ , equal to the ratio of the cross-sectional area of the coils to the cross-sectional area of the corresponding standard and technical documentation [1-3].

For the individual cooling with natural or forced ventilation of low-power instruments, it is usual to use lamellar, finned, pin type, or "crab" type radiators. When the requirements as to the weight of the instrument are stringent, it is recommended to use pin type radiators; when the requirements concern mainly the size, finned radiators are recommended. Louver radiators with forced ventilation are used for cooling medium-power instruments; for groups of low- and medium-power instruments, single-group radiators are used, mostly finned or pin type radiators; they are economically and technologically more advantageous than individual radiators.

Investigation of the heat exchange of various types of radiators made it possible to plot the approximate dependence of the mean superheating  $\vartheta_S = t_S - t_C$  of the base with area  $S_p = L_x \cdot L_y$  on the specific load P/S<sub>p</sub> with natural and forced ventilation (Fig. 1), from which it is then possible to choose the type of radiator and the nature of the heat exchange. The area bounded by the curves  $\alpha_1 - b_1$  pertain to a certain type of radiator with free or forced ventilation. For instance, the area  $\alpha_1 - b_1$  encompasses the values of P/S<sub>p</sub> for different sizes of lamellar radiators with natural ventilation,  $\alpha_4 - b_4$  with forced ventilation, etc.

To characterize the heat exchange properties of radiators, the correlation between the mean superheating  $\vartheta_S$  of the base, the scattered power P, the effective heat transfer coefficient  $\alpha_{ef}$ , the thermal conductivity  $\sigma_{\Sigma}$ , and the thermal resistance  $R_{\Sigma}$  is often used:

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Fig. 1. Graphs for determining the type of radiator and the cooling conditions:  $a_1$ - $b_1$ ,  $a_2$ - $b_2$ ,  $a_3$ - $b_3$ ) lamellar, finned, pin-type radiators, respectively, with free ventilation;  $a_4$ - $b_4$ ) lamellar;  $a_5$ - $b_5$ ) finned;  $a_6$ - $b_6$ ) wire-loop;  $a_7$ - $b_7$ ) louver;  $a_8$ - $b_8$ ) pin-type radiators with forced air motion at speeds V = (2-5) m/sec. ( $t_s - t_c$ ), °K; P/S<sub>p</sub>, W/m<sup>2</sup>.

$$P = \alpha_{\rm ef} \vartheta_S S_{\rm p} = \sigma_{\Sigma} \vartheta_S = \frac{\vartheta_S}{R_{\Sigma}} , \qquad (1)$$

where  $S_p = L_x \cdot L_y$  or  $S_p = \pi D^2/4$ .

All the complexity of the processes of heat exchange and the design features of the radiators are here concentrated in a single magnitude: the effective heat transfer coefficient  $\alpha_{ef}$ , which depends on the type of radiator, the geometric parameters of the finned surfaces, and the nature of the heat exchange, and can be determined experimentally or by calculation. In the former case the procedure is based on dependence (1), which makes it possible to find  $\alpha_{ef}$  from the experimentally found values of P and  $\vartheta_s$  [3].

Designing new radiators and improving existing ones made it necessary to devise a theoretical method of determining the parameters  $\alpha_{ef}$ ,  $\sigma_{\Sigma}$ , and  $R_{\Sigma}$ .

We present the thermal model of a single fin or pin of a radiator in the form of a rod with arbitrary cross section of area f with perimeter v and length H, situated in a medium with temperature  $t_c$ , and having heat transfer coefficient from the lateral surface  $\alpha$ . Dul'nev and Tarnovskii [4] provided an analysis of the heat exchange of such a rod, and showed that the superheating  $\vartheta_i$  of the end face of the i-th rod, into which the heat flux  $P_i$  enters, is determined by the following expression:

$$\vartheta_i = rac{P_i}{\lambda f b} \operatorname{cth} b H'; \ b^2 = rac{lpha v}{\lambda f}; \ H' = H + rac{f}{v}.$$

The parameter b contains the heat transfer coefficient of the lateral surface of the fin or pin, which is determined from the respective criterial dependences [5]. Then the thermal resistance  $R_i$  of a single rod is

$$R_i = \sigma_i^{-1} = \frac{\vartheta_i}{P_i} = \frac{\operatorname{cth} bH'}{\lambda f b} \; .$$

The total conductivity  $\sigma_{\Sigma p}$  of the finned part of the radiator is equal to the sum of the conductivities  $\sigma_i$  of all N fins:

$$\sigma_{\Sigma p} = \sum_{i=1}^{N} \sigma_i = N \sigma_i.$$

If the conductivity from the unfinned part is equal to  $\sigma_{\rm uf},$  then the total conductivity of the radiator is

$$\sigma_{\Sigma} = \sigma_{\rm uf} + N\lambda f b \, {\rm th} \, b H'. \tag{2}$$

Formula (1) makes it possible to determine only the mean temperature of the radiator base. In solving actual problems, it is indispensable to take into account the nonuniformity of the temperature distribution in the base. The thermal model may be represented in the form of a plate with local heat sources of power  $P_i$  and dimensions equal to the dimensions of the elements situated on the base (Fig. 2a). Heat dissipation from the bases proceeds into a medium with temperature  $t_c$ ; we denote the heat transfer coefficient from the finned surface  $\alpha_{ef}$ , from the unfinned surface  $\alpha_{uf}$ .

The equation of thermal conductivity of a rectangular plate with local rectangular heat sources  $2\Delta\xi\Delta\eta$  in size has the form:

$$\frac{\partial^2 \vartheta}{\partial x^2} + \frac{\partial^2 \vartheta}{\partial y^2} - \frac{(\alpha_{\rm ef} + \alpha_{\rm uf})}{\lambda \delta} \quad \vartheta = \Sigma - 1 \{u_i\} - \frac{P_i}{4\Delta \xi \Delta \eta \delta \lambda},$$

$$1 \{u_i\} = \begin{cases} 1 & \text{in the region of source i,} \\ 0 & \text{outside the region of the sources.} \end{cases}$$
(3)

We write the boundary conditions on the end faces of the plate:

$$\left(\frac{\partial \vartheta}{\partial x} \mp \alpha \left|_{\substack{0x\\1x}} \vartheta\right)\right|_{x=\stackrel{0}{\overset{1}{\overset{1}{\overset{1}{\overset{1}{\overset{1}{\overset{1}}{\overset{1}{\overset{1}{\overset{1}}{\overset{1}{\overset{1}}{\overset{1}{\overset{1}}{\overset{1}{\overset{1}}{\overset{1}{\overset{1}}{\overset{1}{\overset{1}}{\overset{1}{\overset{1}{\overset{1}}{\overset{1}{\overset{1}}{\overset{1}{\overset{1}}{\overset{1}{\overset{1}}{\overset{1}{\overset{1}{\overset{1}}{\overset{1}{\overset{1}}{\overset{1}{\overset{1}}{\overset{1}{\overset{1}}{\overset{1}{\overset{1}}{\overset{1}}{\overset{1}{\overset{1}}{\overset{1}{\overset{1}}{\overset{1}{\overset{1}{\overset{1}}{\overset{1}{\overset{1}}{\overset{1}{\overset{1}}{\overset{1}{\overset{1}}{\overset{1}{\overset{1}}{\overset{1}}{\overset{1}{\overset{1}}{\overset{1}}{\overset{1}{\overset{1}}{\overset{1}}{\overset{1}{\overset{1}}{\overset{1}}{\overset{1}{\overset{1}}{\overset{1}{\overset{1}}{\overset{1}{\overset{1}{\overset{1}}{\overset{1}{\overset{1}}{\overset{1}{\overset{1}}{\overset{1}}{\overset{1}{\overset{1}}{\overset{1}}{\overset{1}}{\overset{1}}{\overset{1}{\overset{1}}{\overset{1}}{\overset{1}{\overset{1}}{\overset{1}}{\overset{1}}{\overset{1}{\overset{1}}{\overset{1}}{\overset{1}}{\overset{1}{\overset{1}}{\overset{1}}{\overset{1}{\overset{1}}}{\overset{1}{}}{\overset{1}}}{\overset{1}}{$$

where  $\vartheta = t - t_c$  is the superheating relative to the ambient temperature  $t_c$ ;  $\alpha_{0x}$ ,  $\alpha_{1x}$ ;  $\alpha_{0y}$ ,  $\alpha_{1y}$  are the heat transfer coefficients on the end faces of the plate. The solution of the system (3), (4) was obtained in [6]; it makes it possible to determine the temperature field of the plate, including the temperature at the point of attachment of the elements  $t_m$ .

2. Planning the Design and Selection of the Cooling Regime of a Radiator. When planning or selecting a radiator, the following have to be known: maximal permissible temperature of the working region of the instrument (e.g., of a p-n junction)  $(t_p)_{per}$  or of its housing  $(t_K)_{per}$ ; power  $P_i$  dissipated by the instrument; ambient temperature of the temperature of the incoming flow  $t_c$ ; internal thermal resistance  $R_{in}$  of the instrument between the working region and the housing; method of attaching the instrument to the radiator, which is characterized by the magnitude of the thermal resistance of the contact  $R_K$ .

To ensure a normal thermal regime of a group of semiconductor instruments, the dimensions of the radiator base  $L_x$ ,  $L_y$ , the parameters of the finning, and the nature of the heat exchange (natural or forced) have to be chosen in such a way that the thermal and other technical and economic requirements are satisfied. Planning and design can be effected with the aid of formulas (1), (2), and Fig. 1 by the method of successive approximations.

For solving the problem, we present an arrangement of connection of the thermal resistances between the working region and the environment, as shown in Fig. 2b, from which follows:

$$t_{\rm p} - t_{\rm c} = (t_{\rm p} - t_{\rm R}) + (t_{\rm R} - t_{\rm nl}) + (t_{\rm m} - t_{\rm c}); \ t_{\rm m} - t_{\rm c} = (t_{\rm p} - t_{\rm c}) - P(R_{\rm in} + R_{\rm R}); \ t_{\rm m} - t_{\rm c} = (t_{\rm R} - t_{\rm c}) - PR_{\rm R}.$$
 (5)

 $\vartheta_{S} = \frac{1}{\beta} \left[ (t_{\kappa} - t_{c}) - PR_{\kappa} \right].$ 

Here P is the total power of the elements situated on the radiator.

We introduce the dimensionless magnitude  $\beta$ :

$$\beta = \frac{t_{\rm m} - t_{\rm c}}{\vartheta_{\rm S}} \,. \tag{6}$$

Then we obtain from (5) that

$$\vartheta_{S} = \frac{1}{\beta} [(t_{p} - t_{c}) - P(R_{in} + R_{R})], \qquad (7)$$



Fig. 2. Plate with rectangular heat sources and heat exchange from the bases and end faces (a), radiator 1 with instrument 2 and arrangement of the connection of thermal resistances between the working region of the instrument and the environment (b).



Fig. 3. The dependence  $\beta = f(Bi, S_m/S_p)$ .

Fig. 4. Block diagram of the sequence in designing radiators: input ID; I) determination of  $\vartheta_m = t_m - t_c$ ; II) determination of  $\beta$ , calculation of  $\vartheta_S$ ; III) selection of the type of finning and the nature of the heat exchange; IV) calculation of  $\alpha_{ef}$ ; V) determination of the parameters of the radiator; VI) refined calculation of  $t_{mi}$  and comparison with  $t_m$  per; results of the calculation.

The value of  $\beta$  can be found for any radiator, with any arrangement of the sources, from the solution of the system of equations (3), (4) for any dimensions of the sources. Figure 3 shows a graph for determining  $\beta$  for the central positioning of a single element on the radiator base

$$\beta = f\left(\text{Bi}, \sqrt{\frac{\overline{S_m}}{S_p}}\right)$$
, where  $\text{Bi} = \frac{(\alpha_{\text{ef}} + \alpha_{\text{uf}})}{\lambda \delta} S_p$ 

<u>3. Algorithm for Planning Radiators.</u> Figure 4 shows the block diagram of the sequence in designing radiators. We divide the problem into two stages: at the first stage we choose the nature of the heat exchange, the effective heat transfer coefficient  $\alpha_{ef}$  corresponding to it, and the dimensions of the base  $L_x$ ,  $L_y$ , and  $\delta$ ; from the chosen value of  $\alpha_{ef}$  we find at the second stage the geometric parameters of the finning S, H, d, and we refine the cooling regime (especially the air flow velocity).

At the first stage (block I) the permissible superheating of the radiator base at the place of attachment of the instruments is calculated by formulas (5). Then (block II) it is

necessary to determine by formula (6) the permissible mean superheating of the radiator base  $(t_S - t_c)$ . The coefficient  $\beta$  is determined from Fig. 3; in the first approximation it is assumed that  $\beta = 1.2$ . According to the program in Fig. 1, the type of finning and the nature of the heat exchange (forced or free ventilation) are chosen in block III. The area of the radiator base  $S_p = L_x \cdot L_y$  is chosen on the basis of design considerations. Block IV is the calculation of the heat transfer coefficient  $\alpha_{ef}$  by formula (1).

At the second stage (block V) the known coefficient  $\alpha_{ef}$  is used for determining the geometric parameters of the finning (S, d, H) and the air flow velocity for forced cooling (V). This operation may be carried out by formula (2) by the method of successive approximations or with the aid of the graphs from [3].

In concluding the process of selecting the radiator (block VI) we calculate the temperature of the elements, and for a group of instruments mounted on one base we use the solution of Eq. (3). Correction of the design is carried out according to the suggested block diagram (Fig. 4).

If the elements are mounted on the finned side, the area of the radiator base has to be correspondingly increased. Then we find the geometric and regime parameters (air flow velocity) of the radiator more accurately, and there we endeavor to more fully satisfy the specified technical requirements.

4. Example of Selecting a Radiator for Cooling a Transistor. We have to select a radiator for cooling a transistor which releases a power P = 12 W and is situated inside the block. Contact between the transistor and the radiator is effected over an area  $S_m = 1.97 \cdot 10^{-4} \text{ m}^2$ , the internal thermal resistance of the instrument is  $R_{in} = 0.8 \text{ °K/W}$ , the thermal resistance of the contact is  $R_K = 1.2 \text{ °K/W}$ , the permissible temperature of the collector junction in the transistor is  $(t_p)_{per} = 100 \text{ °C}$ , the conditions of heat exchange are natural convection, and the air temperature in the block  $t_c = 40 \text{ °C}$ .

## Solution.

1. By formula (5) we determine the temperature  $t_m$  at the place of attachment of the transistor  $t_m - t_c = (100 - 40) - 12(0.8 + 1.2) = 36^{\circ}K$ .

2. In the first approximation we adopt  $\beta^{I} = 1.2$ , and from (7) we find that  $\vartheta_{S} = 36/1.2 = 29^{\circ}K$ .

3. On the basis of additional considerations (e.g., the permissible volume in the block for placing a transistor with a radiator) we adopt in the first approximation that the area of the base is  $S_p^{I} = 100 \cdot 10^{-3} \cdot 45 \cdot 10^{-3} = 4.5 \cdot 10^{-3} m^2$ . Then the specific thermal load is

$$q = \frac{P}{S_{\rm p}} = \frac{12}{4.5 \cdot 10^{-3}} = 2.7 \cdot 10^3 \text{ W/m}^2.$$

4. From Fig. 1 and for  $\vartheta_S = 29^{\circ}$ K and  $q = 2.7 \cdot 10^{3}$  W/m<sup>2</sup>, we determine the possible type of finning of the radiator under conditions of natural ventilation. It follows from the illustration that a pin-type radiator has to be chosen.

5. By formula (5) we determine the effective heat exchange coefficient necessary for ensuring the specified thermal regime:

$$\alpha_{\rm ef} = \frac{P}{\vartheta_{\rm S} S_{\rm p}} = \frac{12}{29 \cdot 4.5 \cdot 10^{-3}} = 100 \ {\rm W} / {\rm m}^2 \cdot {}^{\circ}{\rm K}.$$

According to the graphs from [3], the closest finning profile of pin-type radiator corresponds to H =  $32 \cdot 10^{-3}$  m, S =  $7 \cdot 10^{-3}$  m, d =  $2.5 \cdot 10^{-3}$  m, for which  $\alpha_{ef} = 100 \text{ W/m}^2 \cdot ^\circ \text{K}$ .

6. From formula (6) we find the second approximation  $\beta^{II}$ , putting  $\alpha_{ef} = \alpha_{op} + \alpha_{uf}$ , and also selecting the material of the radiator, e.g., duralumin with  $\lambda = 180^{\circ} \text{W/m} \cdot \text{K}$ :

Bi = 
$$\frac{100 \cdot 4.5 \cdot 10^{-3}}{180 \cdot 2.5 \cdot 10^{-3}}$$
 = 1;  $\frac{S_{\rm m}}{S_{\rm n}} = \frac{1.96 \cdot 10^{-4}}{4.5 \cdot 10^{-3}} = 0.044.$ 

From Fig. 3 we find  $\beta^{II} = 1.1$ , and we find a more accurate value of the superheating from formula (7):  $\vartheta_S^{II} = 36/1.1 = 32.7$ °K.

7. We specify more accurately the dimensions of the base and the type of radiator. According to Fig. 1, with  $\vartheta_S = 32.7^{\circ}$ K and  $q = 2.7 \cdot 10^3 \text{ W/m}^2$ , the type of radiator remains as before. We make the base somewhat smaller and find a more accurate value of the parameter q:  $S = 100 \cdot 10^{-3} \cdot 40 \cdot 10^{-3} = 4 \cdot 10^{-3} \text{ m}^2$ ,  $q = 12/(4 \cdot 10^{-3}) = 3 \cdot 10^3 \text{ W/m}^2$ .

By the previous procedure we find the new value  $\beta^{III}$  = 1.1, which means that it is senseless to do any further refining because  $\beta^{II} \approx \beta^{III}$ .

8. Finally, we stipulate a pin-type radiator made of duralumin with base area  $S_p = 4 \cdot 10^{-3} \text{ m}^2$ , H =  $32 \cdot 10^{-3} \text{ m}$ , S =  $7 \cdot 10^{-3} \text{ m}$ , and d =  $2.5 \cdot 10^{-3} \text{ m}$ .

## NOTATION

 $L_x$ ,  $L_y$ , dimensions of the rectangular radiator base; D, diameter of the circular radiator base; H, height of a fin, pin, or turn; d, thickness of fin or wire; S, pitch of finning or helical coiling; S', spacing of the helix;  $\varphi$ , space factor of the channel;  $\vartheta_S$ , mean superheating of the radiator base;  $t_S$ ,  $t_C$ ,  $t_m$ ,  $t_K$ , mean temperature of the radiator base, ambient temperature, temperature under the source on the radiator base, and, temperature of the housing of the semiconductor instrument, respectively; P, power of heat release of an element;  $S_p$ , area of the radiator base;  $S_m$ , area of the instrument;  $\alpha_{ef}$ ,  $\alpha$ , effective and real heat exchange coefficient, respectively;  $\sigma_{\Sigma}$ , conductivity of the radiator;  $R_{\Sigma}$ , resistance; f, cross-sectional area of fin or pin; v, perimeter of fin or pin;  $\lambda$ , thermal conductivity of the material of the radiator; N, number of fins or pins of the radiator;  $\sigma_{uf}$ , conductivity of the unfinned part of the radiator; x, y, coordinates of the i-th source on the radiator base;  $\Delta \xi_i$ ,  $\Delta \eta_i$ , half-size of the source;  $R_{in}$ , internal resistance of the semiconductor instrument;  $R_K$ , contact resistance between instrument housing and radiator;  $\delta$ , thickness of the radiator base.

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